Correlation analysis in automated testing

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3MDEB

Agenda

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- Correlation Matrix
- Use-case
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Introduction

> *Science may be described as the art of systematic over-simplification — the art of discerning what we may with advantage omit.*
>
> ***Karl Popper***

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Who am I

Łukasz Wcisło
Automated Validation Engineer

- automated testing
- platform security
- open-source

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Purpose

For a given aggregation of tests in the validation infrastructure, that does not fulfill the condition of a 100% pass rate exists a correlation coefficient matrix that can be computed and shall describe how often in the past they failed simultaneously.

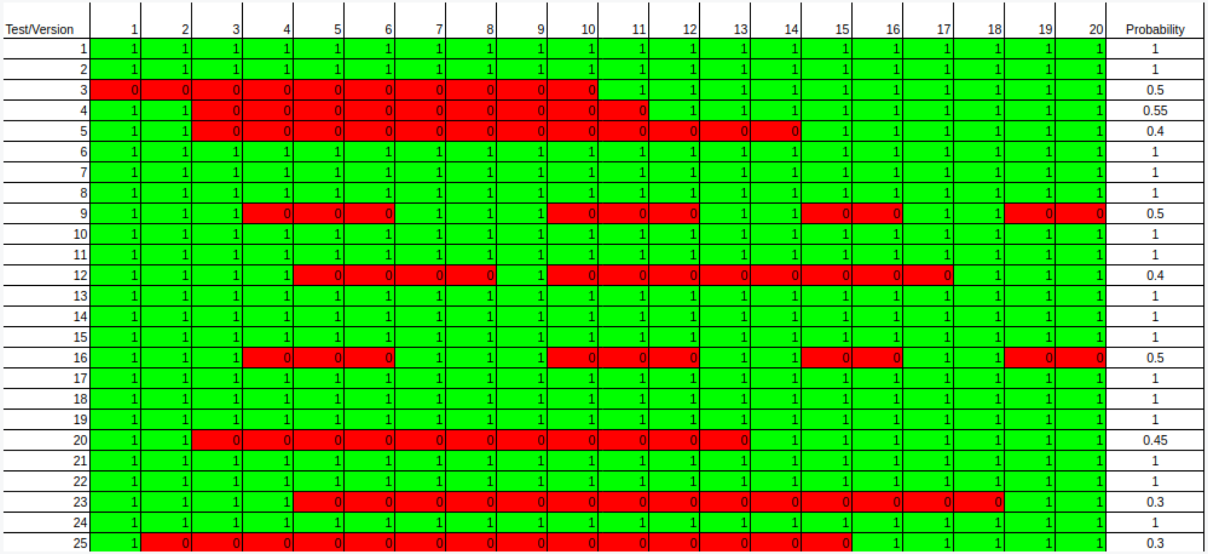
With a given correlation matrix one can optimize validation infrastructure and validate test conditions.

- Simplicity
- Time saving
- Logic
- Elegance

For the perfect test suite there should be no correlation between the tests.

Function definition

Test result as a Boolean function, a relation between a release version and a result of a test.



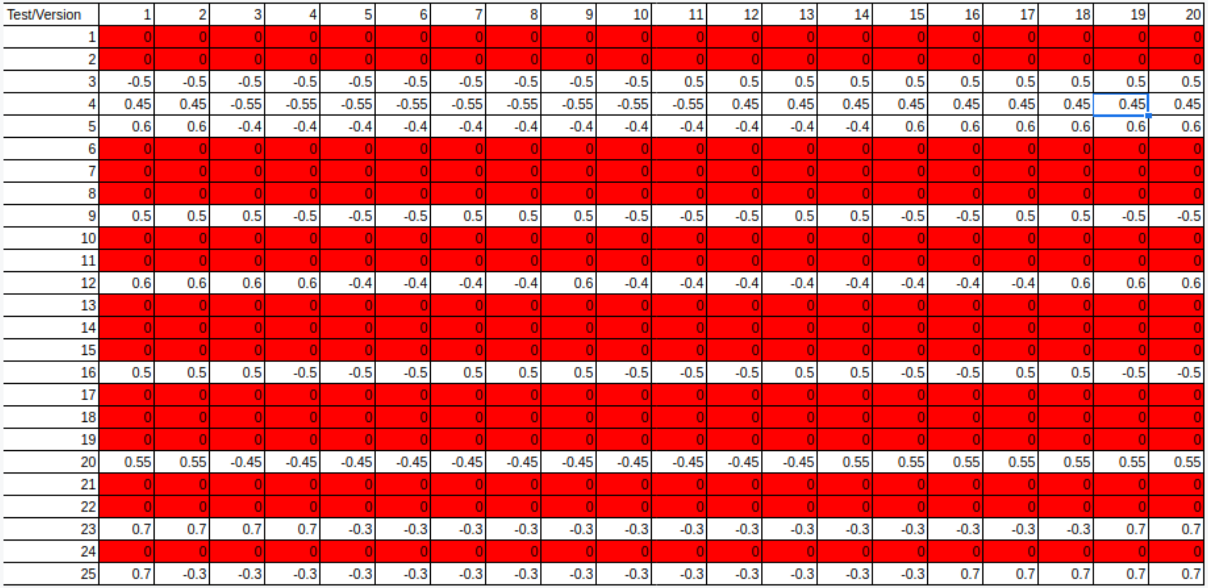
Test/Version	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Probability
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0.5
4	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0.55
5	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0.4
6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9	1	1	1	0	0	0	1	1	1	0	0	0	1	1	0	0	1	1	0	0	0.5
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
12	1	1	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	1	1	1	0.4
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	1	1	1	0	0	0	1	1	1	0	0	0	1	1	0	0	1	1	0	0	0.5
17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20	1	1	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0.45
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
22	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
23	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0.3
24	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
25	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0.3

Red - FAIL

Green - PASS

Function deviations

Instead of using expected value, we can use the probability.



Test/Version	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	-0.5	-0.5	-0.5	-0.5	-0.5	-0.5	-0.5	-0.5	-0.5	-0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
4	0.45	0.45	-0.55	-0.55	-0.55	-0.55	-0.55	-0.55	-0.55	-0.55	-0.55	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45
5	0.6	0.6	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	0.6	0.6	0.6	0.6	0.6	0.6
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0.5	0.5	0.5	-0.5	-0.5	-0.5	0.5	0.5	0.5	-0.5	-0.5	-0.5	0.5	0.5	-0.5	-0.5	0.5	0.5	-0.5	-0.5
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0.6	0.6	0.6	0.6	-0.4	-0.4	-0.4	-0.4	0.6	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	0.6	0.6	0.6
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0.5	0.5	0.5	-0.5	-0.5	-0.5	0.5	0.5	0.5	-0.5	-0.5	-0.5	0.5	0.5	-0.5	-0.5	0.5	0.5	-0.5	-0.5
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0.55	0.55	-0.45	-0.45	-0.45	-0.45	-0.45	-0.45	-0.45	-0.45	-0.45	-0.45	-0.45	0.55	0.55	0.55	0.55	0.55	0.55	0.55
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0.7	0.7	0.7	0.7	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3	0.7	0.7
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0.7	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3	-0.3	0.7	0.7	0.7	0.7	0.7

Covariance matrix

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2n} \\ \vdots & \cdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_n^2 \end{bmatrix}$$

Where

$\sigma_i^2 = D^2 X_i$

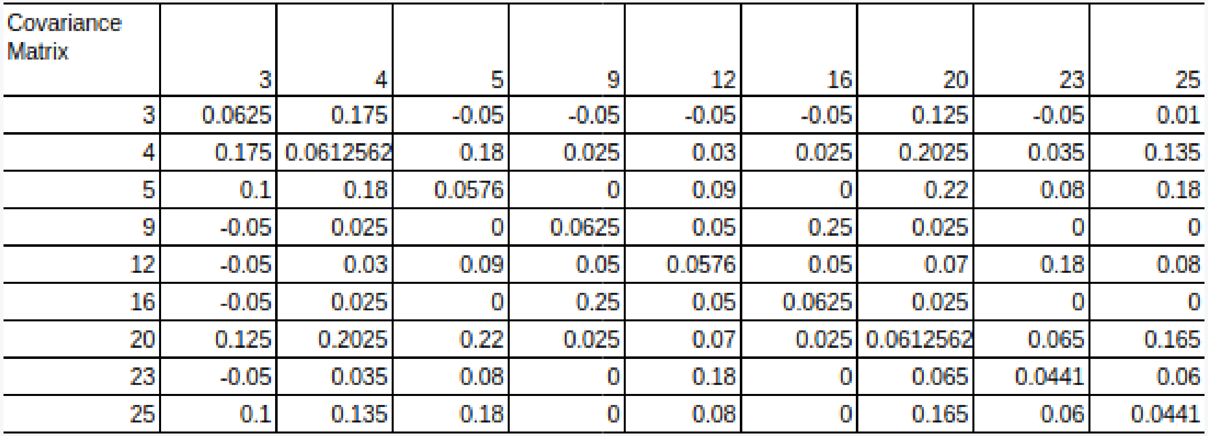
is a variance of variable X, and

$\sigma_{ij} = \text{cov}(X_i, X_j)$

is a covariance between two standardized random variables.

(In our case - between two tests)

Covariance matrix 2



Covariance Matrix	3	4	5	9	12	16	20	23	25
3	0.0625	0.175	-0.05	-0.05	-0.05	-0.05	0.125	-0.05	0.01
4	0.175	0.0612562	0.18	0.025	0.03	0.025	0.2025	0.035	0.135
5	0.1	0.18	0.0576	0	0.09	0	0.22	0.08	0.18
9	-0.05	0.025	0	0.0625	0.05	0.25	0.025	0	0
12	-0.05	0.03	0.09	0.05	0.0576	0.05	0.07	0.18	0.08
16	-0.05	0.025	0	0.25	0.05	0.0625	0.025	0	0
20	0.125	0.2025	0.22	0.025	0.07	0.025	0.0612562	0.065	0.165
23	-0.05	0.035	0.08	0	0.18	0	0.065	0.0441	0.06
25	0.1	0.135	0.18	0	0.08	0	0.165	0.06	0.0441

We can extract meaningful tests for better performance. Diagonal contains variance of each test, covariance matrix is symmetric. Also, every covariance matrix is positive semi-definite.

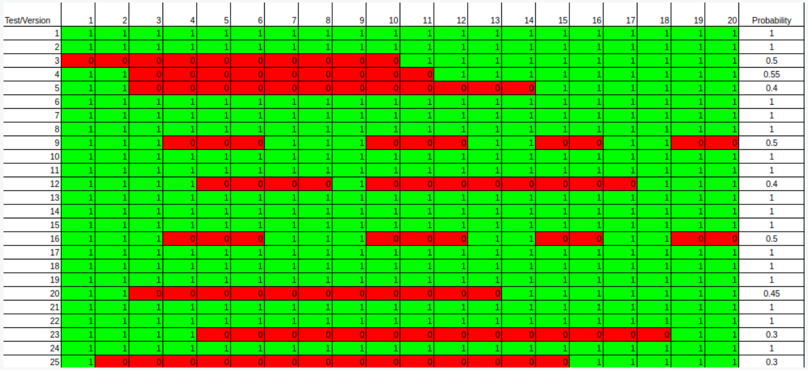
Pearson correlation coefficient

What brings us to Pearson correlation coefficient.

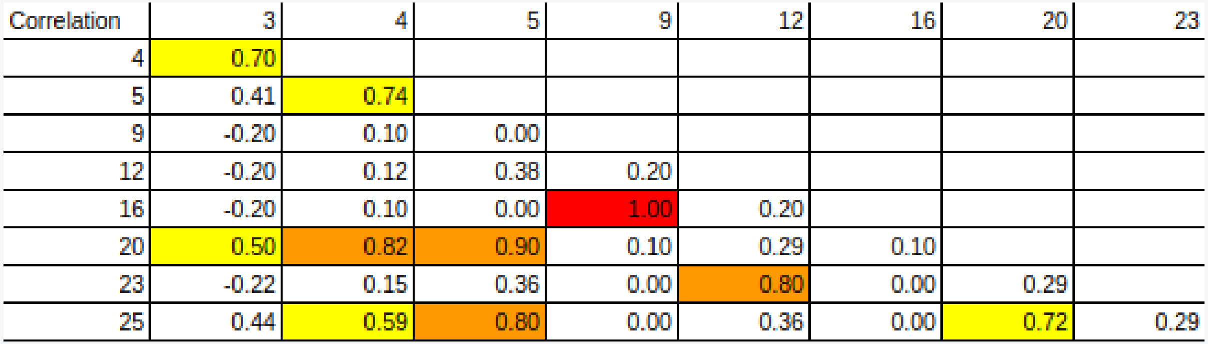
It is a covariance of two variables divided by the product of their standard deviations:

$$r_{XY} = \frac{\operatorname{cov}(X,Y)}{\sigma_X \sigma_Y} = \frac{\left(\sum_{i=1}^{n}\sum_{j=1}^{m} P(X = x_i, Y = y_j) x_i y_j\right) - \overline{X}\,\overline{Y}}{\sqrt{\left(\sum_{i=1}^{n} P(X = x_i) x_i^2\right) - \overline{X}^2}\sqrt{\left(\sum_{i=1}^{m} P(Y = y_i) y_i^2\right) - \overline{Y}^2}}$$

Correlation Matrix



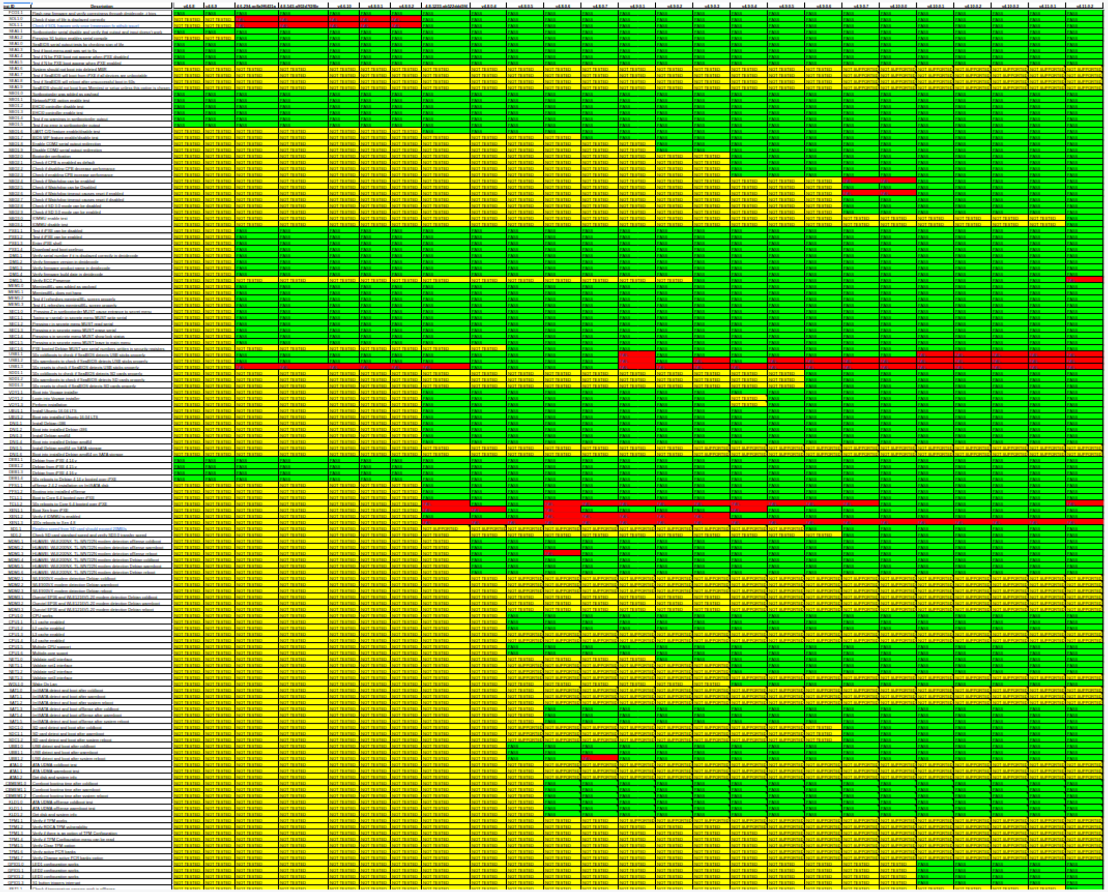
Test/Version	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Probability
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0.5
4	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0.55
5	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0.4
6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9	1	1	1	0	0	0	1	1	1	0	0	0	1	1	0	0	1	1	0	0	0.5
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
12	1	1	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	1	1	1	0.4
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	1	1	1	0	0	0	1	1	1	0	0	0	1	1	0	0	1	1	0	0	0.5
17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20	1	1	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0.45
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
22	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
23	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0.3
24	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
25	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0.3



Correlation	3	4	5	9	12	16	20	23
4	0.70							
5	0.41	0.74						
9	-0.20	0.10	0.00					
12	-0.20	0.12	0.38	0.20				
16	-0.20	0.10	0.00	1.00	0.20			
20	0.50	0.82	0.90	0.10	0.29	0.10		
23	-0.22	0.15	0.36	0.00	0.80	0.00	0.29	
25	0.44	0.59	0.80	0.00	0.36	0.00	0.72	0.29

Where correlation is normalized and always stays between -1 and 1.

Actual use-case

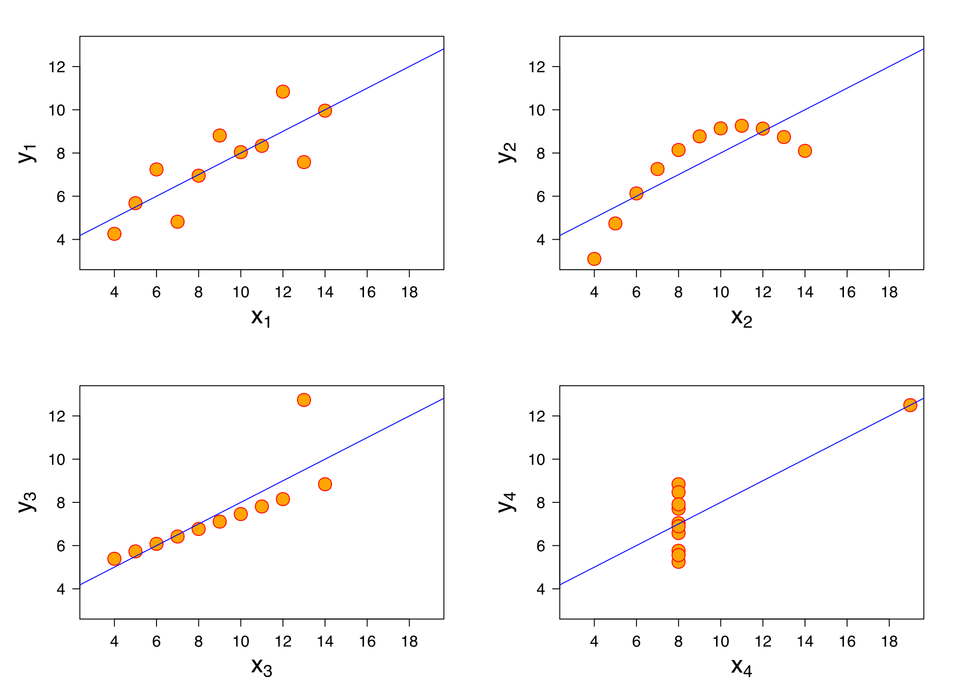


Source: https://docs.google.com/spreadsheets/d/1_uRhVo9eYeZONnelymonYp444zYHT_Q_qmJEJ8_XqJc/edit#gid=0

Conclusion

Basing on the correlation matrix, it can be estimated which tests are PROBABLY not necessary, or have poorly formulated pass conditions. It can have practical use in large test infrastructures, and it can be automated. The most meaningful outcome should be the dynamics of correlation that can be observed during software evolution.

Anscombe’s quartet



Mean of x, of y, variance of x, of y, correlation between x and y, linear regression and coefficient of determination of the linear regression are the same for each data set.

Bibliography

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Q & A

Thank you for your attention

"There are three kinds of lies: lies, damned lies, and statistics."

Benjamin Disraeli